Preference variations in the AK model

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Abstract

The paper shows that the AK model can be solved analytically for Stone-Geary, exponential, and quadratic preferences and that it does not in general exhibit a steady-state solution for Stone-Geary and exponential preferences. The paper also examines preferences in which capital and the change of capital enter into utility (this is especially realistic for human capital). The inclusion of capital in utility typically increases growth, while the presence of disutility from capital investment leads to lower growth.

Keywords: AK model; Capital in utility; Exponential preferences; Quadratic preferences; Stone-Geary preferences

JEL classification: O41

*E-mail: petr.duczynski@uhk.cz. This work was partially supported by the Grant Agency of the Czech Republic, grant number 402/04/0642. This paper was published in the *Bulletin of the Czech Econometric Society*, 11, 20, July 2004, pp. 29-44.

1 Introduction

The AK model is the simplest endogenous growth model. The AK model is basically an endogenous limit of the exogenous Ramsey model. There is just one sector producing consumption goods and there are no diminishing returns to capital. The production function is linear in capital; the level of technology is exogenously given (constant). In the name of this model, the symbol "A" denotes the technological parameter, whereas the symbol "K" corresponds to the capital stock.

Important one-sector extensions of the AK model include Romer (1986) and Barro (1990). Romer (1986) introduces a framework with externalities from physical capital and increasing returns to scale. Barro (1990) discusses a model with government spending entering the production function. In these models, there are no differences in production functions for different types of capital. These differences are introduced, for example, in the Uzawa-Lucas model (see Uzawa, 1965, and Lucas, 1988). In the basic Uzawa-Lucas model, the production of physical capital goods depends on human and physical capital, while the production of human capital is just a function of human capital. Rebelo (1991) and Mulligan and Sala-i-Martin (1993) consider extensions of the Uzawa-Lucas model in which the production of human capital depends both on human and physical capital; the dependence on human capital is, nevertheless, relatively intensive.

The present paper focuses on the one-sector AK model (capital is a composite of human and physical capital). The paper examines the model for different preference structures. The paper first demonstrates that the AK model can be solved analytically for exponential, quadratic, and Stone-Geary preferences. For exponential and Stone-Geary preferences, there are no transitional dynamics in the sense that the solution takes a uniform structure. An important observation is that the solutions for these preferences are not in general steady-state solutions: the growth rates of capital and output are not constant over time. On the other hand, quadratic preferences exhibit transitional dynamics - convergence towards a steady state. The paper then introduces preferences in which instantaneous utility depends on consumption and capital. This setup seems especially plausible for human capital. It is natural to assume that human capital in the form of health and education increases the quality of life for a given amount of consumption. The given framework can be solved analytically for certain preference specifications. We consider specifications for which the steady-state solutions exist. The presence of capital in utility frequently increases the growth rate of the economy. In the end we analyze specifications in which utility depends on consumption, capital, and investment. It is plausible that investment in human capital (learning or physical training) is connected with disutility. This disutility decreases the growth rate of the economy for simple preference specifications. There are no transitional dynamics, but the steady-state solution exists.

2 Consumption in utility

2.1 Stone-Geary preferences

There is no population growth. The problem is

$$\max_{C} \int_0^\infty \frac{(C-\tilde{C})^{1-\theta}-1}{1-\theta} e^{-\rho t} dt$$

subject to

$$\dot{K} = AK - C - \delta K, \tag{1}$$

where C is consumption, $\tilde{C}$ is the subsistence level of consumption, K is the composite of human and physical capital, $\theta > 0$ is a parameter (inverse elasticity of intertemporal substitution if $\tilde{C} = 0$), ρ is the rate of time preference, and δ is the depreciation rate of capital. This is a standard optimization problem in an infinite-horizon framework.[1] The present-value Hamiltonian for this problem is

$$\mathcal{H} = \frac{(C-\tilde{C})^{1-\theta}-1}{1-\theta} e^{-\rho t} + \mu(AK - C - \delta K). \tag{2}$$

The first-order conditions are

$$\mu = e^{-\rho t}(C-\tilde{C})^{-\theta}, \tag{3}$$

$$\dot{\mu} = \mu(\delta - A). \tag{4}$$

The transversality condition is

$$\lim_{t\to\infty} \mu K = \lim_{t\to\infty} K e^{(\delta - A)t} = 0. \tag{5}$$

[1] Infinite-horizon optimal control problems and their applications are discussed, for example, in Takayama (1993).

From the first-order conditions it follows that

$$\dot{C} = \frac{A - \delta - \rho}{\theta}(C - \tilde{C}). \tag{6}$$

The solution to this differential equation is

$$C = \tilde{C} + [C(0) - \tilde{C}]e^{\frac{A-\delta-\rho}{\theta}t}. \tag{7}$$

The differential equation for capital is

$$\dot{K} = (A - \delta)K - \tilde{C} - [C(0) - \tilde{C}]e^{\frac{A-\delta-\rho}{\theta}t}. \tag{8}$$

The general solution for capital is

$$K = \lambda e^{(A-\delta)t} + \frac{\tilde{C}}{A - \delta} + \frac{C(0) - \tilde{C}}{A - \delta - (A - \delta - \rho)/\theta} e^{\frac{A-\delta-\rho}{\theta}t}, \tag{9}$$

where λ is a constant. The transversality condition implies that $\lambda = 0$ and $A - \delta > (A - \delta - \rho)/\theta$. Thus the solution for capital takes the form

$$K = \frac{\tilde{C}}{A - \delta} + \left[K(0) - \frac{\tilde{C}}{A - \delta}\right] e^{\frac{A-\delta-\rho}{\theta}t}. \tag{10}$$

The solution for output is simply an A multiple of K. The growth rate of capital and output is not constant over time unless $\tilde{C} = 0$. Thus the steady-state solution exists only if $\tilde{C} = 0$. The solution takes a uniform structure. In this sense, there are no transitional dynamics.

2.2 Exponential preferences

The problem is

$$\max_{C} \int_0^\infty -\frac{1}{\theta} e^{-\theta C - \rho t} dt$$

subject to (1). The present-value Hamiltonian is

$$\mathcal{H} = -\frac{1}{\theta} e^{-\theta C - \rho t} + \mu(AK - C - \delta K). \tag{11}$$

The first-order conditions are

$$e^{-\theta C - \rho t} = \mu, \tag{12}$$

$$\dot{\mu} = (\delta - A)\mu. \tag{13}$$

The transversality condition takes the same form as in the previous case. From the first-order conditions it follows that

$$C = C(0) + \frac{A - \delta - \rho}{\theta} t. \tag{14}$$

This can be substituted into the equation of motion for capital. The general solution for capital takes the form

$$K = \lambda e^{(A-\delta)t} + \frac{C(0) + (A - \delta - \rho)/[\theta(A - \delta)]}{A - \delta} + \frac{A - \delta - \rho}{\theta(A - \delta)} t. \tag{15}$$

The transversality and initial conditions imply that

$$K = K(0) + \frac{A - \delta - \rho}{\theta(A - \delta)} t. \tag{16}$$

Thus the solution for capital and output takes a linear form. The growth rate of the economy is not constant - the solution is not a steady-state solution.

2.3 Quadratic preferences

The problem is

$$\max_C \int_0^\infty (C - C^2) e^{-\rho t} dt$$

subject to (1). The felicity function is concave. It becomes decreasing for $C > 0.5$. The existence of the bliss point ($C = 0.5$) does not necessarily contradict the economic reality. For example, one can consider the consumption of scoops of ice-cream, for which utility eventually decreases. The present-value Hamiltonian is

$$\mathcal{H} = (C - C^2) e^{-\rho t} + \mu(AK - C - \delta K). \tag{17}$$

The first-order conditions are

$$(1 - 2C) e^{-\rho t} = \mu, \tag{18}$$

$$\dot{\mu} = \mu(\delta - A). \tag{19}$$

From the first-order conditions it follows that

$$C = 0.5 + [C(0) - 0.5] e^{(\delta + \rho - A)t}. \tag{20}$$

This solution is similar to the solution for the Stone-Geary preferences ($\tilde{C} = 0.5$, $\theta = -1$). An important difference is that $C(0)$ must be below $\tilde{C}$ in the present case. Analogously to the Stone-Geary preferences, the solution for capital takes the form:

$$K = \frac{0.5}{A-\delta} + \left[K(0) - \frac{0.5}{A-\delta}\right] e^{(\delta+\rho-A)t}, \tag{21}$$

where $K(0) < \frac{0.5}{A-\delta}$. The economy converges towards its steady state (bliss point) $K = \frac{0.5}{A-\delta}$, $C = 0.5$. The existence of convergence in the AK framework is somewhat surprising. If $K(0) \geq \frac{0.5}{A-\delta}$, it is optimal for the economy to have $C = 0.5$ forever. The solution for capital is then

$$K = \left[K(0) - \frac{0.5}{A-\delta}\right] e^{(A-\delta)t} + \frac{0.5}{A-\delta}. \tag{22}$$

The solution is the same as for the logarithmic Stone-Geary preferences with $\rho = 0$ and $\tilde{C} = 0.5$.

3 Capital in utility

This section considers utility functions depending on consumption and capital. The capital stock in the AK model is a composite of human and physical capital. Regarding human capital, it is quite natural to assume that health and education enter positively into the felicity function. A contribution of health to utility is self evident. A contribution of education to utility deserves some explanation. People would be willing to get educated even if there were no effects of education on income. This situation applied in former communist countries, where returns on education were frequently zero or negative. According to the Soviet statistical yearbook (Narodnoye chozyaystvo SSSR, 1983), the average monthly wages in the U.S.S.R. in 1980 were 127 roubles in the sector of healthcare, physical training, and social security, 136 roubles in the sector of national education, 111 roubles in the sector of culture, 135 roubles in the sector of art, and 180 roubles in the sector of science. In comparison, the average wage of industrial workers (for which the education is plausibly lower - engineers are excluded) was 186 roubles. This is a clear example of negative returns on schooling which can be explained if education is a source of utility.[2]

[2]Filer, Jurajda, and Plánovský (1999) show that returns on education substantially increased during transition in Czech and Slovak economies.

It is likely that human capital increases the productivity of leisure in utility. This assumption has been made by Ortigueira and Santos (1997) in the context of endogenous growth models. I believe that literate people can better enjoy leisure than illiterate people. Some forms of education (such as in the fields of mathematical science, history, or philosophy) are not directly productive in the real economy, but they provide a better orientation in the world, plausibly directly enhancing utility.

Even physical capital can directly contribute to utility. Rich people can sustain high levels of consumption in bad times. Physical capital can be valued for this precautionary reason. Securities provide security - this effect of capital goes beyond the effect on the future expected path of consumption. Alternatively, physical capital can be valued because it provides individuals with an option to make purchases across a variety of goods. The ability to make a choice can be a source of utility even if no purchases are actually realized. Physical capital extends the degree of freedom of individuals. A similar argument applies for human capital - educated people have an option to make a choice over a larger variety of jobs.

Another channel of capital affecting utility is via future expected consumption. The present utility depends not only on the present consumption, but also on the expected future consumption. If I knew that I would win a large sum of money in a lottery 5 years from now, my present utility would be higher even if I faced tight borrowing constraints.

In Duczynski (2001) I examine the effects of the presence of capital in utility in several growth models, including the Uzawa-Lucas model. In that model, the inclusion of human capital in utility tends to increase the steady-state growth rate of the economy, while the presence of physical capital in utility leaves the steady-state growth unchanged.

3.1 Preferences $\frac{(C+\gamma K)^{1-\theta}-1}{1-\theta}$

The problem is

$$\max_{C} \int_0^\infty \frac{(C+\gamma K)^{1-\theta}-1}{1-\theta} e^{-\rho t} dt$$

subject to (1), where $\theta > 0$, $\rho > 0$, and $\gamma > 0$ are parameters. The present-value

Hamiltonian for this problem is

$$\mathcal{H} = \frac{(C+\gamma K)^{1-\theta}-1}{1-\theta}e^{-\rho t} + \mu(AK - C - \delta K). \tag{23}$$

The first-order conditions are

$$(C+\gamma K)^{-\theta}e^{-\rho t} = \mu, \tag{24}$$

$$\dot{\mu} = -\gamma(C+\gamma K)^{-\theta}e^{-\rho t} - \mu(A-\delta). \tag{25}$$

The first-order conditions imply that

$$C + \gamma K = \frac{1}{\mu(0)^{1/\theta}}e^{\frac{A+\gamma-\delta-\rho}{\theta}t}. \tag{26}$$

Thus, the presence of capital in utility (γ) increases the growth rate of the economy. In a steady state, capital and consumption grow at the same rate $\frac{A+\gamma-\delta-\rho}{\theta}$. If the ratio of consumption to capital exceeds the steady-state ratio, the growth rate of consumption is higher than the growth rate of capital. Thus the steady-state solution is the only stable solution. The parameter γ cannot be arbitrarily large to ensure a positiveness of the ratio of consumption to capital.

3.2 Preferences $\frac{C^{1-\theta}-1}{1-\theta} + \gamma\frac{K^{1-\theta}-1}{1-\theta}$

The problem is

$$\max_C \int_0^\infty \left(\frac{C^{1-\theta}-1}{1-\theta} + \gamma\frac{K^{1-\theta}-1}{1-\theta}\right)e^{-\rho t}dt$$

subject to (1). The present-value Hamiltonian is

$$\mathcal{H} = \left(\frac{C^{1-\theta}-1}{1-\theta} + \gamma\frac{K^{1-\theta}-1}{1-\theta}\right)e^{-\rho t} + \mu(AK - C - \delta K). \tag{27}$$

The first-order conditions are

$$C^{-\theta}e^{-\rho t} = \mu, \tag{28}$$

$$\dot{\mu} = -e^{-\rho t}\gamma K^{-\theta} - \mu(A-\delta). \tag{29}$$

From the first-order conditions it follows that

$$\frac{\dot{C}}{C} = \frac{A-\delta-\rho+\gamma(C/K)^{\theta}}{\theta}. \tag{30}$$

In a steady state, the growth rate of consumption is constant and equal to the growth rate of capital. If the ratio of consumption to capital is higher (lower) than the corresponding steady-state ratio, it further increases (decreases). Thus the steady-state solution is the only stable solution. This solution satisfies

$$\frac{A - \delta - \rho + \gamma(C/K)^{\theta}}{\theta} = A - \delta - C/K. \tag{31}$$

C/K depends negatively on γ. Since the steady-state growth rate of the economy equals $A - \delta - C/K$, it depends positively on γ. For logarithmic preferences ($\theta = 1$), the growth rate of the economy turns out to be $A - \delta - \frac{\rho}{1+\gamma}$. The presence of capital in utility increases the growth rate of the economy.

3.3 Preferences $\frac{(C^{1-\nu}K^{\nu})^{1-\theta}-1}{1-\theta}$

The problem is

$$\max_{C} \int_0^{\infty} \frac{(C^{1-\nu}K^{\nu})^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt$$

subject to (1), where ν is a parameter between 0 and 1, and $\theta > 0$ is the inverse of the elasticity of intertemporal substitution of the composite of C and K. The present-value Hamiltonian is

$$\mathcal{H} = \frac{(C^{1-\nu}K^{\nu})^{1-\theta} - 1}{1-\theta} e^{-\rho t} + \mu(AK - \delta K - C). \tag{32}$$

The first-order conditions are

$$e^{-\rho t} K^{\nu(1-\theta)}(1-\nu)C^{(1-\nu)(1-\theta)-1} = \mu, \tag{33}$$

$$\dot{\mu} = -e^{-\rho t} C^{(1-\nu)(1-\theta)} K^{\nu(1-\theta)-1}\nu + \mu(\delta - A). \tag{34}$$

The first-order conditions imply that

$$\frac{\dot{\mu}}{\mu} = -\rho + \nu(1-\theta)\frac{\dot{K}}{K} + [(1-\nu)(1-\theta) - 1]\frac{\dot{C}}{C}, \tag{35}$$

$$\frac{\dot{\mu}}{\mu} = -\frac{\nu}{1-\nu}\frac{C}{K} + \delta - A, \tag{36}$$

$$\frac{\dot{C}}{C}[(1-\nu)(1-\theta) - 1] = \rho - \nu(1-\theta)(A-\delta) + \delta - A + \nu(1-\theta)\frac{C}{K} - \frac{\nu}{1-\nu}\frac{C}{K}. \tag{37}$$

The equation of motion for capital is

$$\frac{\dot{K}}{K} = A - \delta - \frac{C}{K}. \tag{38}$$

The growth rate of C depends positively on C/K, whereas the growth rate of K depends negatively on C/K. If C/K departs from its steady-state value, it further deviates from it. Thus the steady-state solution is the only stable solution. The equality of the growth rates of C and K implies that

$$\frac{C}{K} = \frac{\rho - (1-\theta)(A-\delta)}{\nu/(1-\nu) + \theta}. \tag{39}$$

The right-hand side must be positive and it depends negatively on ν. Thus the growth rate of the economy depends positively on ν.

3.4 Quadratic preferences $C - C^2 + \gamma(K - K^2)$

The problem is

$$\max_C \int_0^\infty [C - C^2 + \gamma(K - K^2)]e^{-\rho t} dt$$

subject to (1). The present-value Hamiltonian is

$$\mathcal{H} = [C - C^2 + \gamma(K - K^2)]e^{-\rho t} + \mu(AK - C - \delta K). \tag{40}$$

The first-order conditions are

$$(1 - 2C)e^{-\rho t} = \mu, \tag{41}$$

$$\dot{\mu} = -\gamma(1 - 2K)e^{-\rho t} - \mu(A - \delta). \tag{42}$$

From these equations we can easily derive that

$$\frac{\dot{\mu}}{\mu} = -\rho - \frac{2C}{1-2C}\frac{\dot{C}}{C}, \tag{43}$$

$$\frac{\dot{\mu}}{\mu} = \delta - A - \gamma\frac{1-2K}{1-2C}. \tag{44}$$

The model's dynamics are greatly complicated. We therefore turn only to the steady-state solution. In the steady state, the growth rates of C, K, and μ are constant. From the equations above it follows that K and C must be constant in the steady state. The steady-state values satisfy:

$$K^* = \frac{A + \gamma - \delta - \rho}{2\gamma + 2(A - \delta - \rho)(A - \delta)}, \tag{45}$$

$$C^* = (A - \delta)K^*. \tag{46}$$

If we assume that $A-\delta < 0.5$ (this is realistic since $A-\delta$ is the return on capital), we have $C^* \leq 0.5$ and $K^* > 0.5$. If $\gamma = 0$, $K^* = \frac{0.5}{A-\delta}$ and $C^* = 0.5$ (consistently with Section 2.3). If $\gamma \to \infty$, $K^* \to 0.5$ and $C^* \to \frac{A-\delta}{2}$. In this model, the presence of capital in utility does not influence the steady-state growth rate of the economy (which is trivially zero). However, it can be shown that γ decreases the values of K^* and C^*. Intuitively, if γ is large, it is optimal to have K^* closer to 0.5 (the bliss point).

3.5 Quadratic preferences $C + \gamma K - (C + \gamma K)^2$

The problem is

$$\max_C \int_0^\infty [C + \gamma K - (C + \gamma K)^2] e^{-\rho t} dt$$

subject to (1). The present-value Hamiltonian for this problem is

$$\mathcal{H} = [C + \gamma K - (C + \gamma K)^2] e^{-\rho t} + \mu(AK - C - \delta K). \tag{47}$$

The first-order conditions are

$$[1 - 2(C + \gamma K)] e^{-\rho t} = \mu, \tag{48}$$

$$\dot{\mu} = [-\gamma + 2(C + \gamma K)\gamma] e^{-\rho t} - \mu(A - \delta). \tag{49}$$

From the first-order conditions it follows that

$$1 - 2(C + \gamma K) = \mu(0) e^{(\delta + \rho - A - \gamma)t}. \tag{50}$$

The right-hand side converges to 0. The corresponding left-hand side then indicates the steady-state solution:

$$K^* = \frac{0.5}{A + \gamma - \delta}, \tag{51}$$

$$C^* = \frac{0.5(A-\delta)}{A+\gamma-\delta} < 0.5. \tag{52}$$

Parameter γ decreases the steady-state values of K^* and C^*. The model's transitional dynamics are described by the following equations:

$$C = 0.5 - \frac{\mu(0)}{2} e^{(\delta+\rho-A-\gamma)t} - \gamma K, \tag{53}$$

$$\dot{K} = (A-\delta+\gamma)K - 0.5 + \frac{\mu(0)}{2} e^{(\delta+\rho-A-\gamma)t}, \tag{54}$$

$$K = \frac{0.5}{A-\delta+\gamma} + \left[K(0) - \frac{0.5}{A-\delta+\gamma}\right] e^{(\delta+\rho-A-\gamma)t}. \tag{55}$$

The presence of capital in utility (parameter γ) speeds up the convergence of the economy towards the steady state.

4 Investment in utility

This section considers preferences in which utility depends on consumption, capital, and the change of capital. Again, this consideration seems especially realistic for human capital. It is plausible that the change of human capital (studying or physical training) is associated with disutility.

The problem is

$$\max_C \int_0^\infty [\ln C + \gamma_1 \ln K + \gamma_2 (A - \delta - C/K)] e^{-\rho t} dt$$

subject to (1). γ_1 and γ_2 are constant parameters (γ_1 is plausibly positive and γ_2 is negative). The present-value Hamiltonian is

$$\mathcal{H} = [\ln C + \gamma_1 \ln K + \gamma_2 (A - \delta - C/K)] e^{-\rho t} + \mu(AK - C - \delta K). \tag{56}$$

The first-order conditions are

$$\frac{e^{-\rho t}}{C} - \frac{e^{-\rho t}\gamma_2}{K} - \mu = 0, \tag{57}$$

$$\dot{\mu} = -\frac{e^{-\rho t}\gamma_1}{K} - \frac{e^{-\rho t}\gamma_2 C}{K^2} + \mu(\delta - A). \tag{58}$$

From this it follows that

$$C = \frac{1}{e^{\rho t}\mu + \gamma_2/K}, \tag{59}$$

$$\frac{\dot{K}}{K} = A - \delta - \frac{1}{e^{\rho t}\mu K + \gamma_2}, \tag{60}$$

$$\frac{\dot{\mu}}{\mu} = \delta - A - \frac{e^{-\rho t}\gamma_1}{\mu K} - \frac{e^{-\rho t}\gamma_2}{\mu K}\frac{1}{e^{\rho t}\mu K + \gamma_2}. \tag{61}$$

Let us introduce a new variable, ψ, such that

$$\psi = \mu K. \tag{62}$$

The transversality condition is

$$\lim_{t\to\infty} \psi = 0. \tag{63}$$

The equation of motion for ψ is

$$\dot{\psi} = -\frac{1+\gamma_1}{e^{\rho t}}. \tag{64}$$

Considering the transversality condition, the solution for ψ is

$$\psi = \frac{1+\gamma_1}{\rho e^{\rho t}}. \tag{65}$$

Thus,

$$C/K = \frac{1}{\gamma_2 + (1+\gamma_1)/\rho}, \tag{66}$$

$$\frac{\dot{K}}{K} = A - \delta - \frac{1}{\gamma_2 + (1+\gamma_1)/\rho}. \tag{67}$$

Thus the only solution is the steady-state solution. The growth rate of the economy is an increasing function of γ_1 and γ_2.

Consider now a similar problem, with the change of capital entering logarithmically in utility. The present-value Hamiltonian is

$$\mathcal{H} = [\ln C + \gamma_1 \ln K + \gamma_2 \ln(A - \delta - C/K)]e^{-\rho t} + \mu(AK - C - \delta K). \tag{68}$$

The first-order conditions are

$$\frac{e^{-\rho t}}{C} - \frac{e^{-\rho t}\gamma_2}{(A-\delta)K - C} - \mu = 0, \tag{69}$$

$$\dot{\mu} = -\frac{e^{-\rho t}\gamma_1}{K} - \frac{e^{-\rho t}\gamma_2 C}{(A-\delta)K^2 - CK} + \mu(\delta - A). \tag{70}$$

From this it follows that

$$\frac{\dot{\mu}}{\mu} = -\frac{e^{-\rho t}(\gamma_1 + 1)}{\mu K} + \frac{C}{K} + \delta - A. \tag{71}$$

If we introduce $\psi = \mu K$, it holds that

$$\dot{\psi} = -\frac{1 + \gamma_1}{e^{\rho t}}. \tag{72}$$

From the transversality condition it follows that

$$\psi = \frac{1 + \gamma_1}{\rho} e^{-\rho t}, \tag{73}$$

$$\frac{1}{C/K} = \frac{1 + \gamma_1}{\rho} + \frac{\gamma_2}{A - \delta - C/K}, \tag{74}$$

$$(C/K)_{1,2} = \frac{\alpha \pm \sqrt{\alpha^2 - 4(1 + \gamma_1)(A - \delta)/\rho}}{2(1 + \gamma_1)/\rho}, \tag{75}$$

where

$$\alpha = 1 + \gamma_2 + (1 + \gamma_1)(A - \delta)/\rho. \tag{76}$$

From the previous case we know that $C/K = \frac{\rho}{1+\gamma_1}$ if $\gamma_2 = 0$. Thus, a negative sign holds in equation (75). I examined the monotonicity of C/K in γ_1 and γ_2 if $A = 0.1$, $\delta = 0.05$, and $\rho = 0.02$. In this calibration, C/K is decreasing both in γ_1 and in γ_2. Thus the growth rate of the economy is increasing in γ_1 and γ_2. The presence of capital in utility increases growth, while the presence of disutility from capital investment leads to lower growth. There are no transitional dynamics. The steady-state solution is the only solution.

5 Conclusion

This paper considers the AK endogenous growth model for various preference specifications. For Stone-Geary preferences, the solution for capital and output is an exponential function of time plus a constant. This constant is zero only if there are preferences with a constant elasticity of intertemporal substitution (zero subsistence level of consumption). Only in this case a steady-state solution exists. For exponential preferences, the solution for capital and output is a linear function of time. Again, the solution is not a steady-state solution. For Stone-Geary and exponential preferences, the solutions take uniform structures. In this sense there are no transitional dynamics. On the other hand, quadratic preferences result exponential convergence towards a steady state (bliss point).

In addition, the paper considers some preference structures in which utility depends positively on consumption and capital. Human capital in the form of health and education plausibly increases utility. Even physical capital can directly contribute to utility by providing a certain level of freedom and security. The inclusion of capital in utility frequently increases the growth rate of the economy.

In the end the paper examines very simple preference structures in which utility depends on consumption, capital, and the time derivative of capital (investment). The inclusion of investment in utility is plausible for human capital - studying or training decreases utility. The presence of capital in utility increases the growth rate of the economy, while the presence of disutility from a change in capital decreases the growth rate.

References

Barro, R.J., 1990, Government spending in a simple model of endogenous growth, Journal of Political Economy 98, S103-S125.

Duczynski, P., 2001, Models of growth with capital contributing to utility, manuscript, Economics Institute, Prague, Czech Republic.

Filer, R.K., S. Jurajda, and J. Plánovský, 1999, Education and wages in the Czech and Slovak Republics during transition, Labour Economics 6, 581-593.

Lucas, R.E., 1988, On the mechanics of economic development, Journal of Monetary Economics 22, 3-42.

Mulligan, C.B. and X. Sala-i-Martin, 1993, Transitional dynamics in two-sector models of endogenous growth, Quarterly Journal of Economics 108, 737-773.

Narodnoye chozyaystvo SSSR v 1982 g. (National Economy of the U.S.S.R. in 1982), 1983, Finansy i statistika, Moscow, U.S.S.R.

Ortigueira, S. and M.S. Santos, 1997, On the speed of convergence in endogenous growth models, American Economic Review 87, 383-399.

Rebelo, S., 1991, Long-run policy analysis and long-run growth, Journal of Political Economy 99, 500-521.

Romer, P.M., 1986, Increasing returns and long-run growth, Journal of Political Economy 94, 1002-1037.

Takayama, A., 1993, Analytical Methods in Economics, University of Michigan Press.

Uzawa, H., 1965, Optimal technical change in an aggregative model of economic growth, International Economic Review 6, 18-31.