Asymmetric Adjustment Costs and the Imbalance Effect between Human and Physical Capital

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Abstract

Empirical studies indicate a positive dependence of the output growth on the ratio of human capital to physical capital (an imbalance effect). This paper considers standard one-capital and two-capital one-sector endogenous and exogenous growth models with adjustment costs. Two-capital models exhibit an empirically plausible imbalance effect between human and physical capital if adjustment costs are larger for human than for physical capital.

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1 Introduction

Whether the growth rate of output is related positively or negatively to the ratio of human capital to physical capital is an important problem in the theory and empirics of economic growth. Casual empiricism favors a positive dependence of growth on the ratio of human to physical capital. Postwar West Germany and Japan had high ratios of human to physical capital[1] and really grew rapidly in the 1950s and 1960s. Maddison (1991) provides evidence for relatively strong capital deepening in Western Europe and Japan between 1950 and 1987 (as compared to the United States). Transition economies in Central and Eastern Europe are in some respects similar to postwar economies. These economies lack physical capital, but they have relatively educated labor force. A number of these countries achieve high output growth rates today. It is important to realize that a significant inflationary bias makes the measured growth rates in these countries underestimated (see Filer and Hanousek, 2000). Transition economies have experienced substantial improvements in the quality of goods; these improvements are not fully taken into account in the measured inflation; with too high inflation, the actual real output growth is higher than the measured one. On the other hand, economies with low human-physical ratios, such as economies after the Black Death in the Middle Ages, did not tend to grow fast (see Hirshleifer, 1987, Chapters 1 and 2).

Cross-country regressions yield similar results. Barro (1991) and Barro and Sala-i-Martin (1995, Chapter 12) show that the growth of output per capita is positively correlated with initial human capital if one controls for the initial level of output and for other variables. Elsewhere (Duczynski, 2003) I examine a sample of 73 countries and conclude that the growth of output per capita between 1960 and 1990 depended significantly positively on the initial ratio of average years of schooling to physical capital per capita. This result is connected with the fact that initial human-physical ratios were remarkably high in Asian tigers (Indonesia, Japan, Korea, Singapore, Taiwan, and Thailand).

The theory of economic growth has ambiguous implications concerning the relationship between the output growth and the human-physical ratio. One-sector

[1] Krug (1967) shows that the ratio of *physical capital* to a measure of human capital fell from approximately 4.0 in 1938 to roughly 2.6 in 1949 in West Germany.

endogenous growth models with human and physical capital and irreversibility restrictions for gross investment predict a U-shaped dependence of growth on the ratio of human to physical capital: the growth is fast if the initial human-physical ratio is significantly below or above its steady-state value (see Barro and Sala-i-Martin, 1995, Chapter 5). A similar prediction applies for the growth rate of output of physical goods in two-sector endogenous growth models [the classic examples of these models were developed by Uzawa (1965) and Lucas (1988)]. However, for a broad concept of output (if the value of gross investment in human capital is added to the production of physical goods), the growth rate depends positively on the human-physical ratio (see Mulligan and Sala-i-Martin, 1993, and Barro and Sala-i-Martin, 1995, Chapter 5).[2] A similar asymmetric imbalance effect between human and physical capital can occur in a model with large adjustment costs for human-capital investment. Adjustment costs for human capital are plausibly large - the process of learning fundamentally takes time. In Duczynski (2002) I examine a one-sector open-economy exogenous growth model with large adjustment costs for human capital and small adjustment costs for physical capital. I show that the growth rate of output per effective worker depends positively on the ratio of human to physical capital in the model's log-linearized version. The model is relatively complicated - it involves a system of four differential equations.[3]

The present paper introduces asymmetric adjustment costs in an open-economy endogenous growth framework and in a closed-economy exogenous growth framework. The selection of models is dictated by tractability. The endogenous growth models considered are tractable in an open-economy setup. We want to keep the analysis as simple as possible - for this reason we examine the standard AK model and its natural two-capital extension. Unlike the endogenous models, the Solow-Swan exogenous growth model does not work well if applied to open economies.[4]

[2]In Duczynski (2004) I demonstrate that the imbalance effect in the Uzawa-Lucas model becomes less plausible if there is a low physical capital share in the physical capital sector (such as 1/3). In particular, the model is implausible for narrow output.

[3]There is one equation for human capital, one for physical capital, and two equations for corresponding q variables.

[4]Barro and Sala-i-Martin (1995, Chapter 3) and Duczynski (2002) examine a Ramsey-type open-economy neoclassical growth model with endogenous investment decisions and adjustment costs. It would probably be very difficult to introduce adjustment costs in an open-economy Solow-Swan model with exogenous investment decisions.

For this reason we focus on closed-economy exogenous models. Again, the Solow-Swan model is a standard model, and we keep the analysis simple.

The paper first considers an open-economy AK model with adjustment costs. The model can be solved analytically; there are no transitional dynamics; the steady-state solution is the only stable solution. The problem is well defined only for some parameter values. The paper then extends this model by assuming two types of capital - human and physical capital - with larger adjustment costs for human than for physical capital. The model can be log-linearized around the steady state and leads to a system of three differential equations. In this respect, the model is somewhat simpler than the exogenous growth model discussed in Duczynski (2002).[5] The model's dynamics involve one negative eigenvalue. For plausible parameter specifications, the absolute value of this eigenvalue (the speed of convergence) exceeds 10%. This speed of convergence is much higher than the speed of convergence in closed-economy neoclassical growth models with broadly viewed capital (2%), as well as than the speed of convergence in two-sector closed-economy endogenous growth models with realistically high adjustment costs (where the speed of convergence is again around 2%, see Ortigueira and Santos, 1997).[6] It can be proven that the growth rate of output depends positively on the human-physical ratio if the adjustment-cost parameter is higher for human than for physical capital.

The paper also considers the Solow-Swan closed-economy neoclassical growth model with adjustment costs. One-capital and two-capital cases are examined. As expected, the speed of convergence is negatively related to adjustment-cost parameters. In the two-capital model, the dynamics of the log-linearized model involve two negative eigenvalues. The model is stable. The growth rate of output depends on the human-physical ratio and on the output level. The dependence on the human-physical ratio is positive if adjustment costs are larger for human

[5]The present model is basically an endogenous limit of the model analyzed in Duczynski (2002). I believe that it is still of some interest to separately examine this endogenous limit. In some sense endogenous growth models are fundamentally different from exogenous growth models.

[6]Ortigueira and Santos (1997) find that the speed of convergence is high in a relatively broad class of endogenous growth models with no adjustment costs. Alvarez Albelo (1999) shows that the speed of convergence in the Uzawa-Lucas model is reduced if there is *learning by using*, i.e., if the change of human capital depends positively on the change of physical capital.

than for physical capital. This is a theoretical framework for the Barro regressions, which included both output and human capital as important explanatory variables.

2 Endogenous growth models

2.1 An AK model

Barro and Sala-i-Martin (1992) briefly examine a closed-economy version of the AK model with adjustment costs. Here we assume that the economy is fully open - it faces the world real interest rate, r. Production depends linearly on capital:

$$Y = AK, \tag{1}$$

where Y is output, A is a fixed technological parameter, and K is the capital stock. Firms solve the investment problem (they maximize the discounted stream of dividends), which can be written in the aggregate form:

$$\max_{I} \int_0^\infty e^{-rt}\Big[AK - I\Big(1 + \frac{b}{2}\frac{I}{K}\Big)\Big]dt$$

subject to

$$\dot{K} = I - \delta K, \tag{2}$$

where $b > 0$ is the adjustment-cost parameter, I is gross investment, and $\delta > 0$ is the depreciation rate of capital. A dot over K denotes a time derivative of K. We assume that the net return on capital, $A - \delta$, is higher than the world interest rate r (this condition can be satisfied in the equilibrium because investment is associated with adjustment costs). The current-value Hamiltonian is

$$\mathcal{J} = AK - I - \frac{b}{2}\frac{I^2}{K} + q(I - \delta K), \tag{3}$$

where q is the marginal shadow value of K (Tobin's q).[7] The first-order conditions are

$$q = 1 + b\frac{I}{K}, \tag{4}$$

[7] Hayashi (1982) shows that marginal q equals average q if firms are price-takers with constant returns to scale in both production and installation.

$$\frac{\dot{q}}{q} = r + \delta - \frac{A + (q-1)^2/(2b)}{q}. \tag{5}$$

The growth rate of the economy is given by

$$g = \frac{\dot{K}}{K} = \frac{q-1}{b} - \delta. \tag{6}$$

The transversality condition is

$$\lim_{t\to\infty} e^{-rt} qK = 0. \tag{7}$$

The transversality condition says that the asymptotic growth rate of the economy is less than r, which can be rewritten as

$$q^* < 1 + (r+\delta)b, \tag{8}$$

where q^* is the steady-state value of q. This value is given by

$$q^* = 1 + (r+\delta)b - \sqrt{(r+\delta)^2 b^2 + 2b(r+\delta) - 2Ab}, \tag{9}$$

where the minus sign before the square root is dictated by the transversality condition. The value of q^* is well defined only for some values of A. The steady-state growth rate of the economy, g^*, satisfies

$$g^* = r - \frac{\sqrt{(r+\delta)^2 b^2 + 2b(r+\delta) - 2Ab}}{b}. \tag{10}$$

The steady-state solution is the only stable solution. The equation of motion for q is

$$\dot{q} = (r+\delta)q - A - (q-1)^2/(2b). \tag{11}$$

The derivative of the right-hand side of this equation with respect to q is positive at $q = q^*$. Consequently, there are no convergent paths to $q = q^*$ other than $q(t) = q^*$ for any t. Equation (6) indicates that a fixed value of q is associated with a fixed value of g ($g = g^*$ for $q = q^*$). There are no economically reasonable transitional dynamics.

2.2 A model with two capital types

Let the production function be given by

$$Y = AK^\alpha H^{1-\alpha}, \tag{12}$$

where K is physical capital, H is human capital, and $0 < \alpha < 1$ is the share of physical capital. The investment problem is analogous to the problem in the one-capital model:

$$\max_{I_K, I_H} \int_0^\infty e^{-rt} \left[AK^\alpha H^{1-\alpha} - I_K \left(1 + \frac{b_K}{2} \frac{I_K}{K} \right) - I_H \left(1 + \frac{b_H}{2} \frac{I_H}{H} \right) \right] dt$$

subject to

$$\dot{K} = I_K - \delta K, \tag{13}$$

$$\dot{H} = I_H - \delta H, \tag{14}$$

where I_K is gross investment in physical capital, I_H is gross investment in human capital, b_K is the adjustment-cost parameter for physical capital, and b_H is the adjustment-cost parameter for human capital. The current-value Hamiltonian is

$$\mathcal{K} = AK^\alpha H^{1-\alpha} - I_K - \frac{b_K}{2} \frac{I_K^2}{K} - I_H - \frac{b_H}{2} \frac{I_H^2}{H} + q_K (I_K - \delta K) + q_H (I_H - \delta H). \tag{15}$$

The first-order conditions are

$$q_K = 1 + b_K \frac{I_K}{K}, \tag{16}$$

$$q_H = 1 + b_H \frac{I_H}{H}, \tag{17}$$

$$\frac{\dot{q}_K}{q_K} = r + \delta - \frac{A\alpha \mathcal{H}^{1-\alpha} + (q_K - 1)^2/(2b_K)}{q_K}, \tag{18}$$

$$\frac{\dot{q}_H}{q_H} = r + \delta - \frac{A(1-\alpha)\mathcal{H}^{-\alpha} + (q_H - 1)^2/(2b_H)}{q_H}, \tag{19}$$

where

$$\mathcal{H} = \frac{H}{K}. \tag{20}$$

The transversality conditions are

$$\lim_{t \to \infty} e^{-rt} q_K K = 0, \tag{21}$$

$$\lim_{t \to \infty} e^{-rt} q_H H = 0. \tag{22}$$

These transversality conditions are equivalent to $r > g^*$, where g^* is the steady-state growth rate of the economy (the common growth rate of human capital, physical capital, and output). The equation of motion for $\mathcal{H}$ is

$$\frac{\dot{\mathcal{H}}}{\mathcal{H}} = \frac{\dot{H}}{H} - \frac{\dot{K}}{K} = \frac{q_H - 1}{b_H} - \frac{q_K - 1}{b_K}. \tag{23}$$

The steady-state growth rate of $\mathcal{H}$ is 0. The steady state is characterized by

$$q_K^*\left(r+\delta-\frac{g^*+\delta}{2}\right)=A\alpha\mathcal{H}^{*1-\alpha}-\frac{g^*+\delta}{2}, \tag{24}$$

$$q_H^*\left(r+\delta-\frac{g^*+\delta}{2}\right)=A(1-\alpha)\mathcal{H}^{*-\alpha}-\frac{g^*+\delta}{2}. \tag{25}$$

These formulae follow from the first-order conditions if we take into account that

$$\frac{q_K^*-1}{b_K}=\frac{q_H^*-1}{b_H}=g^*+\delta. \tag{26}$$

The equations of motion for $\mathcal{H}$, q_K, and q_H can be log-linearized around the steady state:

$$\begin{pmatrix} d\ln(\mathcal{H}/\mathcal{H}^*)/dt \\ d\ln(q_K/q_K^*)/dt \\ d\ln(q_H/q_H^*)/dt \end{pmatrix}=\begin{pmatrix} 0 & \mathcal{B} & \mathcal{C} \\ \mathcal{D} & \mathcal{E} & 0 \\ \mathcal{F} & 0 & \mathcal{E} \end{pmatrix}\begin{pmatrix} \ln(\mathcal{H}/\mathcal{H}^*) \\ \ln(q_K/q_K^*) \\ \ln(q_H/q_H^*) \end{pmatrix}, \tag{27}$$

where

$$\mathcal{B}=-\frac{q_K^*}{b_K}<0, \tag{28}$$

$$\mathcal{C}=\frac{q_H^*}{b_H}>0, \tag{29}$$

$$\mathcal{D}=-\frac{A\alpha}{q_K^*}(1-\alpha)\mathcal{H}^{*1-\alpha}<0, \tag{30}$$

$$\mathcal{E}=r-g^*>0, \tag{31}$$

$$\mathcal{F}=\frac{A\alpha}{q_H^*}(1-\alpha)\mathcal{H}^{*-\alpha}>0. \tag{32}$$

The Jacobian matrix of the system has two positive and one negative eigenvalues. Consequently, the model is saddle-path stable. To satisfy the transversality conditions, the dynamics of the system are driven only by the negative eigenvalue. The absolute value of this eigenvalue is the speed of convergence of the system. The negative eigenvalue is given by

$$\epsilon=\frac{\mathcal{E}-\sqrt{\mathcal{E}^2+4(\mathcal{B}\mathcal{D}+\mathcal{C}\mathcal{F})}}{2}. \tag{33}$$

The dynamics of $\mathcal{H}$ are determined by

$$\ln(\mathcal{H}/\mathcal{H}^*)=\ln[\mathcal{H}(0)/\mathcal{H}^*]e^{\epsilon t}, \tag{34}$$

where $\mathcal{H}(0)$ is the initial value of $\mathcal{H}$. The dynamics of Tobin's q's depend on eigenvectors corresponding to the eigenvalue ϵ:

$$\ln(q_K/q_K^*) = \ln(\mathcal{H}/\mathcal{H}^*)\frac{A\alpha(1-\alpha)\mathcal{H}^{*1-\alpha}}{q_K^*(\mathcal{E}-\epsilon)}, \tag{35}$$

$$\ln(q_H/q_H^*) = \ln(\mathcal{H}/\mathcal{H}^*)\frac{-A\alpha(1-\alpha)\mathcal{H}^{*-\alpha}}{q_H^*(\mathcal{E}-\epsilon)}. \tag{36}$$

The critical question is how the growth rate of output in this model depends on the ratio of human to physical capital. The growth rate of output, g, can be written as

$$g = \alpha\frac{q_K-1}{b_K} + (1-\alpha)\frac{q_H-1}{b_H} - \delta, \tag{37}$$

which can be approximated around the steady state:

$$g \approx g^* + \alpha\frac{q_K^*}{b_K}\ln(q_K/q_K^*) + (1-\alpha)\frac{q_H^*}{b_H}\ln(q_H/q_H^*), \tag{38}$$

$$g \approx g^* + \frac{A\alpha(1-\alpha)^2}{\mathcal{E}-\epsilon}\mathcal{H}^{*-\alpha}\left[\frac{\mathcal{H}^*\alpha}{(1-\alpha)b_K} - \frac{1}{b_H}\right]\ln(\mathcal{H}/\mathcal{H}^*). \tag{39}$$

It is natural to assume that $b_H > b_K$. Unlike physical capital, which can be installed relatively quickly, human-capital investment (learning) fundamentally takes time. It is very difficult to significantly increase the level of knowledge or experience in a short period of time.[8] Parameters b_K and b_H satisfy

$$b_K = \frac{A\alpha\mathcal{H}^{*1-\alpha} - (r+\delta)}{(g^*+\delta)[r+\delta-(g^*+\delta)/2]}, \tag{40}$$

$$b_H = \frac{A(1-\alpha)\mathcal{H}^{*-\alpha} - (r+\delta)}{(g^*+\delta)[r+\delta-(g^*+\delta)/2]}. \tag{41}$$

These equations result from (24)-(26).

Lemma 1: If $b_H > b_K$, then $\mathcal{H}^*\frac{\alpha}{(1-\alpha)b_K} > \frac{1}{b_H}$. If $b_H = b_K$, then $\mathcal{H}^*\frac{\alpha}{(1-\alpha)b_K} = \frac{1}{b_H}$.

Proof: From (40) and (41) it follows that

$$\mathcal{H}^*\frac{\alpha}{(1-\alpha)b_K} - \frac{1}{b_H} = \frac{a_1a_2}{a_3a_4}, \tag{42}$$

where

$$a_1 = (\delta+g^*)\left(r+\delta-\frac{g^*+\delta}{2}\right)(r+\delta), \tag{43}$$

[8] A similar assumption appears, for example, in Barro and Sala-i-Martin (1995, Chapters 3 and 5), Obstfeld and Rogoff (1996, p. 461), and Duczynski (2002).

$$a_2 = \frac{(1-\alpha)/\alpha}{\mathcal{H}^*} - 1, \tag{44}$$

$$a_3 = (1-\alpha)A\mathcal{H}^{*-\alpha} - \frac{(r+\delta)(1-\alpha)/\alpha}{\mathcal{H}^*}, \tag{45}$$

$$a_4 = (1-\alpha)A\mathcal{H}^{*-\alpha} - (r+\delta). \tag{46}$$

The transversality conditions imply that $a_1 > 0$. Since $b_K > 0$ and $b_H > 0$, it must hold that $a_3 > 0$ and $a_4 > 0$. If $b_H > b_K$, it is $a_2 > 0$ [see equations (40) and (41)]. If $b_H = b_K$, it is $a_2 = 0$, Q.E.D.

Proposition 1: If $b_H > b_K$, the growth rate of output depends positively on the ratio of human capital to physical capital. If $b_H = b_K$, the growth rate of output is independent of the human-physical ratio and equals g^*.

Proof: It follows directly from Lemma 1, equation (39), and the fact that $\mathcal{E} > 0$ and $\epsilon < 0$, Q.E.D.

The speed of convergence exceeds 10% for realistic parameter specifications.[9] For example, let us consider $A = 0.25$, $g^* = 0.02$, $r = 0.06$, $\alpha = 0.3$, and $\delta = 0.05$. If $\mathcal{H}^*\alpha/(1-\alpha) = 1$, then $q_K^* = q_H^* = 1.34$, and $\epsilon = -0.148$. In this calibration, the steady-state rate of return on physical capital $\frac{\partial Y}{\partial K} - \delta$ is 8.6%. The rate of return on human capital $\frac{\partial Y}{\partial H} - \delta$ is the same. If $\mathcal{H}^*\alpha/(1-\alpha) = 0.9$, then $q_K^* = 1.21$, $q_H^* = 1.40$, and $\epsilon = -0.171$. The rates of return are 7.6% on physical capital and 9.0% on human capital. If $\mathcal{H}^*\alpha/(1-\alpha) = 0.8$, then $q_K^* = 1.08$, $q_H^* = 1.47$, and $\epsilon = -0.257$. The rates of return are 6.6% on physical capital and 9.5% on human capital. Because of adjustment costs, all the given rates of return exceed r. The steady-state rate of return on human capital is higher than the rate of return on physical capital if adjustment costs are larger for human than for physical capital ($q_H^* > q_K^*$, or, equivalently, $b_H > b_K$). The speed of convergence, $|\epsilon|$, tends to increase with larger differences in adjustment costs between human and physical capital. The speed of convergence is close to values found for two-sector endogenous growth models without adjustment costs (around 20%, see Ortigueira and Santos, 1997). On the other hand, Ortigueira and Santos observe that convergence is slow in two-sector endogenous growth models with reasonably high adjustment costs (the speed of convergence is then around 2%). Thus we observe large differences in the speed of convergence between one-sector

[9] Caselli et al. (1996) use a generalized method of moments estimator and find the convergence speed of approximately 10 percent per year.

and two-sector endogenous growth models with adjustment costs.[10] Similarly, the observed speed of convergence is much larger than the speed of convergence in the closed-economy neoclassical growth model with broadly viewed capital (around 2%).[11]

3 Exogenous growth models

3.1 The Solow-Swan model with one type of capital

We now consider a fully closed economy. Saving decisions are exogenous; the economy saves a constant fraction of output. For the model without adjustment costs, see Solow (1956), Swan (1956), and Barro and Sala-i-Martin (1995, Chapter 1). We assume that the loss associated with adjustment costs is a quadratic function of saving and that it depends negatively on the existing capital. Investment equals saving less losses associated with capital installation. [Barro and Sala-i-Martin (1995) also mention the Solow-Swan model with adjustment costs. In their framework, adjustment costs depend on investment (rather than saving). Their results do not differ substantially from the present results if adjustment costs are small.] There are diminishing returns to capital, so that output per effective worker, y, depends on capital per effective worker, k, in the following way:

$$y = k^\alpha, \tag{47}$$

where $0 < \alpha < 1$ is the capital share. The equation of motion for capital is

$$\dot{k} = sk^\alpha - \frac{b}{2}\frac{(sk^\alpha)^2}{k} - (\delta + n + x)k, \tag{48}$$

where $s > 0$ is the saving rate, $b > 0$ is the adjustment-cost parameter, $\delta > 0$ is the depreciation rate of capital, n is the population growth rate, and x is the growth rate of technology. These parameters are exogenously determined. The steady-state capital per effective worker, k^*, satisfies

$$sk^{*\alpha-1} - \frac{b}{2}s^2k^{*2(\alpha-1)} = \delta + n + x. \tag{49}$$

[10] Unlike the present model, Ortigueira and Santos (1997) develop a closed-economy framework.

[11] The speed of convergence is high in the neoclassical growth model if capital is not broadly defined (see King and Rebelo, 1993).

The presence of b decreases the steady-state level of capital. If we introduce the following substitution:

$$z = sk^{*\alpha-1}, \tag{50}$$

we get

$$z_{1,2} = \frac{1 \pm \sqrt{1 - 2b(\delta + n + x)}}{b}. \tag{51}$$

If $b = 0$, $z = \delta + n + x$. Thus a minus sign in the equation for z holds. The equation of motion for capital can be log-linearized around the steady state:

$$\frac{\dot{k}}{k} = -\beta \ln(k/k^*), \tag{52}$$

where the speed of convergence, β, is given by

$$\beta = (1-\alpha)(z - bz^2), \tag{53}$$

$$\beta = (1-\alpha)\Big[2(\delta + n + x) + \frac{1}{b}\Big(\sqrt{1 - 2b(\delta + n + x)} - 1\Big)\Big]. \tag{54}$$

The equation of motion for output is analogous to the equation for capital:

$$\frac{\dot{y}}{y} = -\beta \ln(y/y^*) \tag{55}$$

Coefficient β is decreasing in b.[12] For small values of b, β can be approximated as

$$\beta \approx (1-\alpha)\Big[\delta + n + x - \frac{1}{2}b(\delta + n + x)^2\Big]. \tag{56}$$

The result from Barro and Sala-i-Martin (1995) is

$$\beta_{BS} = (1-\alpha)(x + n + \delta)\frac{1 + b(x + n + \delta)/2}{1 + b(x + n + \delta)}. \tag{57}$$

This result can be approximated the same way as β.

3.2 The Solow-Swan model with two types of capital

Production depends on physical and human capital:

$$y = k^{\alpha}h^{\eta}, \tag{58}$$

[12] See the proof of Lemma 2 in the subsequent section.

where $\alpha > 0$, $\eta > 0$, and $\alpha + \eta < 1$. Analogously to the previous section, the equations of motion for both capital types are

$$\dot{k} = s_K k^{\alpha} h^{\eta} - \frac{b_K}{2} \frac{(s_K k^{\alpha} h^{\eta})^2}{k} - (\delta + n + x)k, \tag{59}$$

$$\dot{h} = s_H k^{\alpha} h^{\eta} - \frac{b_H}{2} \frac{(s_H k^{\alpha} h^{\eta})^2}{h} - (\delta + n + x)h, \tag{60}$$

where s_K is the saving rate for physical capital, and s_H is the saving rate for human capital; b_K and b_H are adjustment-cost parameters. Let us introduce the notation similar to the previous section:

$$z_K = s_K k^{*\alpha-1} h^{*\eta}, \tag{61}$$

$$z_H = s_H k^{*\alpha} h^{*\eta-1}. \tag{62}$$

It holds that

$$z_K = \frac{1 - \sqrt{1 - 2b_K(\delta + n + x)}}{b_K}, \tag{63}$$

$$z_H = \frac{1 - \sqrt{1 - 2b_H(\delta + n + x)}}{b_H}, \tag{64}$$

$$k^{*1-\alpha-\eta} = s_K^{1-\eta} s_H^{\eta} z_K^{\eta-1} z_H^{-\eta}, \tag{65}$$

$$h^{*1-\alpha-\eta} = s_H^{1-\alpha} s_K^{\alpha} z_H^{\alpha-1} z_K^{-\alpha}. \tag{66}$$

The equations of motion can be log-linearized around the steady state:

$$\frac{\dot{k}}{k} = \frac{d\ln(k/k^*)}{dt} = \mathcal{P}\ln(k/k^*) + \mathcal{Q}\ln(h/h^*), \tag{67}$$

$$\frac{\dot{h}}{h} == \frac{d\ln(h/h^*)}{dt} = \mathcal{R}\ln(k/k^*) + \mathcal{S}\ln(h/h^*), \tag{68}$$

where

$$\mathcal{P} = (\alpha - 1)\Omega_K < 0, \tag{69}$$

$$\mathcal{Q} = \eta\Omega_K > 0, \tag{70}$$

$$\mathcal{R} = \alpha\Omega_H > 0, \tag{71}$$

$$\mathcal{S} = (\eta - 1)\Omega_H < 0, \tag{72}$$

where $\Omega_K = z_K - b_K z_K^2$ and $\Omega_H = z_H - b_H z_H^2$.

Lemma 2: Ω_K is decreasing in b_K, and Ω_H is decreasing in b_H.

Proof: It suffices to consider the dependence of Ω_K on b_K (the case of Ω_H is analogous). It holds that

$$\Omega_K = \frac{\sqrt{1-2b_K(\delta+n+x)}}{b_K} - \frac{1}{b_K} + 2(\delta+n+x). \tag{73}$$

The derivative of the right-hand side with respect to b_K is negative if and only if

$$\frac{(\delta+n+x)b_K}{\sqrt{1-2b_K(\delta+n+x)}} + \sqrt{1-2b_K(\delta+n+x)} > 1. \tag{74}$$

The left-hand side is positive. If we make a squared power of both sides, this inequality turns out to be equivalent to

$$\frac{(\delta+n+x)^2 b_K^2}{1-2b_K(\delta+n+x)} > 0, \tag{75}$$

which is trivially satisfied. Thus, Ω_K decreases in b_K, Q.E.D.

Corollary: If $b_H > b_K$, then $\Omega_K > \Omega_H$.

Lemma 3: $\Omega_K \geq 0$ and $\Omega_H \geq 0$. $\Omega_K = 0$ only for $b_K = \frac{1}{2(\delta+n+x)}$, and $\Omega_H = 0$ only for $b_H = \frac{1}{2(\delta+n+x)}$.

Proof: The second part of this Lemma follows directly if we substitute for b_K in (73). The uniqueness is ensured by the monotonicity of Ω_K in b_K. The first part results from the monotonicity of Ω_K in b_K and the fact that $b_K \leq \frac{1}{2(\delta+n+x)}$, Q.E.D.

The Jacobian matrix of the system (67) and (68) has two negative eigenvalues:

$$\lambda_{1,2} = \frac{\mathcal{P}+\mathcal{S} \pm \sqrt{(\mathcal{P}+\mathcal{S})^2 - 4(\mathcal{P}\mathcal{S}-\mathcal{Q}\mathcal{R})}}{2} \tag{76}$$

The solution for physical and human capital per effective worker is

$$\ln(k/k^*) = \nu_1 e^{\lambda_1 t} + \nu_2 e^{\lambda_2 t}, \tag{77}$$

$$\ln(h/h^*) = \nu_1 \frac{\lambda_1 - \mathcal{P}}{\mathcal{Q}} e^{\lambda_1 t} + \nu_2 \frac{\lambda_2 - \mathcal{P}}{\mathcal{Q}} e^{\lambda_2 t}, \tag{78}$$

where coefficients ν_1 and ν_2 are determined by initial conditions for k and h:

$$\nu_1 = \frac{\mathcal{Q}\ln[h(0)/h^*] - (\lambda_2 - \mathcal{P})\ln[k(0)/k^*]}{\lambda_1 - \lambda_2}, \tag{79}$$

$$\nu_2 = \frac{-\mathcal{Q}\ln[h(0)/h^*] + (\lambda_1 - \mathcal{P})\ln[k(0)/k^*]}{\lambda_1 - \lambda_2} \tag{80}$$

The solution for h results from the solution for k if we use components of eigenvectors corresponding to eigenvalues λ_1 and λ_2. The solution for output per effective worker is

$$\ln(y/y^*) = \nu_1\Big(\alpha + \eta\frac{\lambda_1 - \mathcal{P}}{\mathcal{Q}}\Big)e^{\lambda_1 t} + \nu_2\Big(\alpha + \eta\frac{\lambda_2 - \mathcal{P}}{\mathcal{Q}}\Big)e^{\lambda_2 t}. \tag{81}$$

Negative eigenvalues λ_1 and λ_2 ensure the stability of the system. Output converges towards its steady state. The evolution of output depends on initial values of physical and human capital.

It is useful to examine the dynamics of the model in terms of the imbalance between human and physical capital. Analogously to Section 2.2, we introduce

$$\mathcal{H} = h/k, \tag{82}$$

$$\mathcal{H}^* = h^*/k^*. \tag{83}$$

Equations (67) and (68) imply that

$$\frac{\dot{k}}{k} = \frac{\eta\Omega_K}{\alpha+\eta}\ln(\mathcal{H}/\mathcal{H}^*) - \frac{(1-\alpha-\eta)\Omega_K}{\alpha+\eta}\ln(y/y^*), \tag{84}$$

$$\frac{\dot{h}}{h} = -\frac{\alpha\Omega_H}{\alpha+\eta}\ln(\mathcal{H}/\mathcal{H}^*) - \frac{(1-\alpha-\eta)\Omega_H}{\alpha+\eta}\ln(y/y^*). \tag{85}$$

Lemma 3 indicates that the growth rate of both types of capital depends practically negatively on $\ln(y/y^*)$, hence also on y. This is a standard effect of diminishing returns. If the ratio of human to physical capital exceeds its steady-state value, it contributes positively to the growth of physical capital and negatively to the growth of human capital. Any initial imbalance between human and physical capital tends to diminish over time. This can be best seen if we write the equation of motion for $\mathcal{H}$:

$$\frac{\dot{\mathcal{H}}}{\mathcal{H}} = \frac{\dot{h}}{h} - \frac{\dot{k}}{k} = -\frac{\alpha\Omega_H + \eta\Omega_K}{\alpha+\eta}\ln(\mathcal{H}/\mathcal{H}^*) + \frac{1-\alpha-\eta}{\alpha+\eta}(\Omega_K - \Omega_H)\ln(y/y^*) \tag{86}$$

If $\mathcal{H} > \mathcal{H}^*$, the imbalance term tends to decrease $\mathcal{H}$. Similarly, if $\mathcal{H} < \mathcal{H}^*$, the imbalance term increases $\mathcal{H}$. If $b_H > b_K$, the corollary of Lemma 2 implies that the growth of $\mathcal{H}$ depends positively on $\ln(y/y^*)$, hence also on y. If, for example, $\mathcal{H} = \mathcal{H}^*$ and $y < y^*$, $\mathcal{H}$ starts to decrease, but y converges to y^*. Eventually the decline of $\mathcal{H}$ turns to an increase, and $\mathcal{H}$ converges back to $\mathcal{H}^*$.

The following is the most important part of this section, which includes the dependence of the growth of output per effective worker, g, on $\mathcal{H}$ and y:

$$g = \alpha\frac{\dot{k}}{k} + \eta\frac{\dot{h}}{h} = \frac{\alpha\eta}{\alpha+\eta}(\Omega_K - \Omega_H)\ln(\mathcal{H}/\mathcal{H}^*) - \frac{1-\alpha-\eta}{\alpha+\eta}(\alpha\Omega_K + \eta\Omega_H)\ln(y/y^*). \tag{87}$$

Proposition 2: If $b_H > b_K$, the growth rate of y depends positively on $\mathcal{H}$.

Proof: It follows directly from equation (87) and the corollary of Lemma 2, Q.E.D.

Proposition 3: Unless $b_K = b_H = \frac{1}{2(\delta+n+x)}$, the growth rate of y depends negatively on y.

Proof: It follows directly from equation (87) and Lemma 3, Q.E.D.

The term $\frac{1-\alpha-\eta}{\alpha+\eta}(\alpha\Omega_K+\eta\Omega_H)$ can be interpreted as the convergence coefficient. It is equal to the partial derivative of g with respect to $\ln(y/y^*)$ (while keeping $\mathcal{H}$ fixed). This term would equal $(1-\alpha-\eta)(\delta+n+x)$ if there were no adjustment costs ($b_K = b_H = 0$). Since Ω_K decreases in b_K and Ω_H decreases in b_H (Lemma 2), the convergence coefficient decreases with larger adjustment costs. This is quite an intuitive result, and it is similar to the result in the previous section.

Equation (87) and Propositions 2 and 3 state that there are two sources of transitional dynamics - the imbalance effect between human and physical capital [the term $\ln(\mathcal{H}/\mathcal{H}^*)$], and the effect of diminishing returns [the term $\ln(y/y^*)$]. This result is similar to Duczynski (2002), where I examined an exogenous Ramsey-type open-economy model with endogenous investment decisions. We see from the present model that endogenous saving (investment) decisions are not necessary for the imbalance effect; this effect is just a natural result of asymmetric adjustment costs. It is, however, likely that the imbalance effect would be stronger if we had endogenous investment decisions. The simultaneous dependence of the output growth on the human-physical ratio and the output level seems to provide a rigorous rationale for the Barro regressions - the growth regressions which include both human capital and output as important explanatory variables (see, for example, Barro, 1991, and Barro and Sala-i-Martin, 1995). In these regressions, the output growth typically depends positively on initial human capital and negatively on the initial output level. Additional explanatory variables (e.g., political stability, openness, market distortions, terms of trade) are frequently used to account for the steady state.

An interesting property of the present model is possible overshooting the steady state. This follows from equation (87). If the output starts just below its steady state and $\mathcal{H} \gg \mathcal{H}^*$, the output increases above the steady state, and after that it converges back to the steady state.

4 Conclusion

The dependence of the output growth on the ratio of human capital to physical capital (the imbalance effect) is an important problem in the theory and empirics of economic growth. To address this issue, this paper examines standard open-economy one-sector endogenous growth models and closed-economy one-sector exogenous growth models with adjustment costs for capital investment. The selection of models is dictated by tractability - for this reason the models are relatively simple.

An open-economy AK model with adjustment costs can be solved analytically in its precise version. There are no transitional dynamics; the steady-state solution is the only solution. An open-economy endogenous growth model with two types of capital (a natural extension of the AK model) can be solved analytically in its log-linear approximation. The open-economy setup makes the model more tractable. The dynamics of the model involve one negative eigenvalue; its absolute value is the speed of convergence. Since there are two other positive eigenvalues, the model is saddle-path stable. The speed of convergence exceeds 10% for realistic parameter specifications. If adjustment costs are larger for human than for physical capital, the growth rate of output depends positively on the ratio of human to physical capital. This is an empirically relevant result.

Exogenous growth models with exogenous saving decisions and adjustment costs can be solved in their log-linear approximations. The given models work well only in a closed-economy setup. The speed of convergence depends negatively on adjustment-cost parameters. The dynamics are based on one negative eigenvalue in the one-capital model and two negative eigenvalues in the two-capital model. Thus, the models are stable. Again, the two-capital model leads to an empirically plausible imbalance effect between human and physical capital if adjustment costs are larger for human capital than for physical capital. In this case the output growth depends positively on the human-physical ratio and negatively on the level of output. This result provides a rigorous framework for the regression studies which include both human capital and output as relevant explanatory variables (the Barro regressions).

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