Adjustment Costs in a Neoclassical Model with Capital Mobility

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Abstract

The paper analyzes the convergence behavior of the open-economy neoclassical growth model with one type of capital. Capital investment is assumed to be subjected to the costs of adjustment. The dependence of the convergence coefficient on the capital share, the elasticity of substitution between labor and capital, and the adjustment-cost specification is examined.

Keywords: adjustment costs; capital mobility; convergence; neoclassical growth

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1 Introduction

The basic neoclassical growth model[1] works relatively well if applied to closed economies with no capital mobility. If the capital share is sufficiently high, which is

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[1]See the classic papers by Cass [5], Koopmans [12], Ramsey [16], Solow [17], and Swan [18].

realistic if capital is viewed broadly (consisting of physical as well as human components), the model predicts slow conditional convergence that is consistent with empirical observations (see, for example, Mankiw et al. [14]).

The assumption of no capital mobility is, however, problematic. This assumption seems especially unrealistic for regions within countries. Moreover, international capital mobility is relatively high today and a large number of developing countries are strongly influenced by global capital markets.

It is well understood that the neoclassical growth model with perfect capital mobility exhibits several problematic properties. As a result of diminishing returns on capital, the model implies infinitely rapid capital flows from rich to poor economies,[2] equalization of the rates of return across all open economies, and immediate convergence of capital and output to their steady-state levels. These predictions are, of course, clearly counterfactual. Although capital flows are important for a large number of economies, the magnitudes of these flows are still relatively small when compared with the GDP levels. Similarly, the convergence speed of open economies seems to be relatively low.[3]

Barro and Sala-i-Martin [1], Chapter 3, propose several modifications of the open-economy neoclassical growth model. These modifications include borrowing restrictions, finite horizons, and adjustment costs for investment.

In the framework with borrowing restrictions (Barro et al. [2], Cohen and Sachs [7]), open credit-constrained economies tend to converge just slightly faster than closed economies. Nevertheless, this framework still implies immediate adjustment for unconstrained economies if there are no costs associated with capital installation. Since the world's net external asset position is zero, there is as much lending as borrowing; thus, some countries are clearly unconstrained. Moreover, it is hard to believe that borrowing constraints are binding for many regional economies such as the U.S. states, Japanese prefectures, or European regions. It is plausible that capital is highly mobile across regions within the same country; one can consequently expect regional economies to be credit constrained only if the net indebtedness of

[2] Strictly speaking, this direction of capital flows results if technological differences are not large between rich and poor economies.

[3] Barro and Sala-i-Martin [1], Chapters 11 and 12, find that both regions within countries (more open economies) and individual countries (less open economies) converge conditionally at the rate of about 2% per year.

these economies is large.[4]

In addition, the assumption of binding credit constraints may require certain pre-specified ratios of different types of capital and foreign debt (as is the case in the model of Barro et al. [2]). If there are no irreversibility restrictions, credit-constrained economies may converge only from below to their steady states.[5]

The present paper develops a model in which borrowing restrictions are replaced by adjustment costs for capital investment. The model extends the analysis of Barro and Sala-i-Martin [1], Chapter 3, by assuming more general adjustment-cost and production-function specifications. The adjustment-cost model is relatively tractable in the absence of borrowing restrictions because of the separability of consumption and investment decisions.

Attention is paid to investigating how the rate of convergence can be reduced if the unit adjustment costs are strictly convex rather than linear, if the composite capital share is high, and if labor and capital are good substitutes in production. The analysis basically applies the quantitative approach, implemented, for example, by King and Rebelo [10] in a closed-economy framework. The model leads to a relatively large spectrum of convergence coefficients. Sufficiently slow convergence (e.g., at the rate of 2% per year) is not automatic, but it can be achieved for certain parameter values if the adjustment costs for changing composite capital are assumed to be reasonably high (plausibly as a result of large adjustment costs for the human component of capital).

The remainder of the paper is organized as follows. Section 2 develops the model, including the optimization problems of firms. Section 3 derives the log-linear approximation. Section 4 introduces the main definitions of the convergence coefficient. Section 5 discusses the Cobb-Douglas and the CES production specifications. In Section 6, the convergence behavior is illustrated for several parameter specifications. Section 7 concludes the paper.

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[4]Elsewhere (Duczynski [8]) I provide evidence that borrowing contraints are not likely to be binding for most economies.

[5]In reality, many economies may be converging from above if their steady states are low (e.g., as a consequence of low technology levels, unfavorable policies, or high population growth rates) as compared with their current levels of GDP per effective capita. Cho and Graham [6] find that low income countries tend to converge from above.

2 The Model

The standard neoclassical growth model with adjustment costs is adopted.[6] Assume a small open economy consisting of N_F firms. The economy can borrow or lend capital on the world credit market at a constant real rate, r. Let the production of the i-th firm ($i = 1, 2, ..., N_F$) exhibit constant returns to scale (CRTS) and positive and diminishing returns for both inputs:

$$Y_i = F_i(K_i, L_i e^{xt}), \tag{1}$$

where Y_i is the firm's output; K_i and L_i are capital (composite of human and physical capital) and labor employed, respectively. Technological progress is labor-augmenting at a constant, exogenous rate, x. The production function is assumed to be identical for all firms. The equation of motion for capital is

$$\dot{K}_i = I_i - \delta K_i, \tag{2}$$

where I_i is the firm's gross investment, and δ is the depreciation rate. The i-th firm starts with the initial capital stock

$$K_i(0) = K_{i0}. \tag{3}$$

Assume that each firm faces costs (adjustment costs) associated with capital installation. Let us follow the standard assumption that these costs depend positively on the level of investment relative to the level of existing capital. The unit installation cost is thus assumed in the form $\phi(I_i/K_i)$ with $\phi(0) = 0$, $\phi'(.) > 0$, and $\phi''(.) \geq 0$.

2.1 Optimization of Firms

The objective of each firm is to maximize the present discounted value of its dividends

$$\max_{I_i, L_i} \int_0^\infty e^{-rt}\{F_i(K_i, L_i e^{xt}) - wL_i - I_i[1 + \phi(I_i/K_i)]\}dt,$$

subject to (2) and (3), where w denotes the wage rate paid on the labor employed. The current-value Hamiltonian for this problem is

$$\mathcal{M}_i = F_i(K_i, L_i e^{xt}) - wL_i - I_i[1 + \phi(I_i/K_i)] + q_i[I_i - \delta K_i],$$

[6] See, for example, Barro and Sala-i-Martin [1], Section 3.5, or Blanchard and Fischer [3], Section 2.4. These examples follow the classic paper by Hayashi [9].

where q_i is the marginal shadow value of K_i. The first-order conditions are

$$\frac{\partial \mathcal{M}_i}{\partial I_i} = -[1 + \phi(I_i/K_i) + I_i/K_i\phi'(I_i/K_i)] + q_i = 0 \tag{4}$$

$$\frac{\partial \mathcal{M}_i}{\partial L_i} = \frac{\partial F_i}{\partial L_i} - w = 0, \tag{5}$$

$$\dot{q}_i = rq_i - \frac{\partial \mathcal{M}_i}{\partial K_i} = (r+\delta)q_i - \frac{\partial F_i}{\partial K_i} - (I_i/K_i)^2\phi'(I_i/K_i). \tag{6}$$

The transversality condition is

$$\lim_{t\to\infty} e^{-rt} q_i K_i = 0. \tag{7}$$

The transversality condition is a boundary condition imposed on the asymptotic behavior of the capital stock. For more information concerning transversality conditions, see, for example, Michel [15].

Equation (5) says that the marginal product of labor is the same across all firms. This condition implies the equalization of the marginal products of capital across all firms as long as the production function is the same for all firms and exhibits CRTS (which is the case here). Now that $\frac{\partial F_i}{\partial K_i}$ is independent of i, the equilibrium paths of I_i/K_i and q_i are also independent of i [note that if $K_i(t)$ and $q_i(t)$ solve the problem for a given initial condition K_{i0}, then $\xi K_i(t)$ and $q_i(t)$ solve the same problem for the initial condition ξK_{i0}, where ξ is a real positive number]. The time evolution of the aggregate capital stock in the economy is therefore independent of the particular distribution of capital among firms.[7]

2.2 The Intensive Form

We have shown the aggregability of the problem. Thus, the problem can be rewritten in the intensive form (in terms of the variables per unit of effective labor). Let $L = \sum_{i=1}^{N_F} L_i$ be the aggregate labor. Let $y = \sum_{i=1}^{N_F} Y_i/(Le^{xt})$, $k = \sum_{i=1}^{N_F} K_i/(Le^{xt})$, and $i = \sum_{i=1}^{N_F} I_i/(Le^{xt})$. Further, let q denote the common value of q_i. The equations (1), (2), (3), (4), (6), and (7), take the following forms:

$$y = F(k,1) = f(k), \tag{8}$$

[7]In the present setup, all investment decisions are made by firms. This is appropriate for physical capital, but perhaps less realistic for human capital. However, the result would remain unchanged if the investment decisions were made at the household level.

$$\dot{k} = i - (\delta + n + x)k, \tag{9}$$

$$k(0) = k_0, \tag{10}$$

$$q = 1 + \phi(i/k) + (i/k)\phi'(i/k), \tag{11}$$

$$\dot{q} = (r + \delta)q - [f'(k) + (i/k)^2\phi'(i/k)], \tag{12}$$

$$\lim_{t\to\infty} e^{-(r-x-n)t} qk = 0, \tag{13}$$

Equation (8) follows from constant returns to scale of the production function. Equation (9) follows from the fact that L grows at the rate of n. Equation (11) results from equation (4); the ratio of investment to capital is the same for all firms. From (11), it is possible to express i/k in terms of q:[8]

$$\frac{i}{k} = \Psi(q). \tag{14}$$

Substituting (14) in (9) and (12) yields

$$\frac{\dot{k}}{k} = \Psi(q) - (\delta + n + x), \tag{15}$$

$$\frac{\dot{q}}{q} = r + \delta - [f'(k) + \Psi^2(q)\phi'(\Psi(q))]/q. \tag{16}$$

¿From (15), it follows that q is constant in the steady state. Then, from (16), k must be also constant in the steady state. Let q^* and k^* denote the steady-state values of q and k, respectively. Equation (13) then requires $r - x - n > 0$, which is a condition on the exogenous parameters r, x, and n.

2.3 Adjustment Costs Specification

It is difficult to solve the system (15) – (16) for a general form of the function $\phi(.)$. Let us simplify the analysis by assuming

$$\phi(i/k) = \frac{b}{\omega + 1}(i/k)^{\omega}, \tag{17}$$

where $\omega \geq 1$ and $b > 0$. The problem takes the following form:

$$q = 1 + b(i/k)^{\omega}, \tag{18}$$

$$\Psi(q) = [(q-1)/b]^{1/\omega}, \tag{19}$$

[8] Note that (11) implies a monotonic relationship between q and i/k.

$$\frac{\dot{k}}{k} = [(q-1)/b]^{1/\omega} - (x+n+\delta), \tag{20}$$

$$\frac{\dot{q}}{q} = r + \delta - [f'(k) + (q-1)^{(1+1/\omega)} b^{-1/\omega} \omega/(1+\omega)]/q. \tag{21}$$

The steady-state values, q^* and k^*, satisfy

$$q^* = 1 + b(x+n+\delta)^{\omega}, \tag{22}$$

$$f'(k^*) = (r+\delta)q^* - (q^*-1)^{(1+1/\omega)} b^{-1/\omega} \omega/(1+\omega). \tag{23}$$

3 Log-Linearization

Using Taylor's expansion, we can log-linearize equations (20) and (21) around the steady state:

$$\frac{\dot{k}}{k} = \frac{d\ln(k/k^*)}{dt} = \mathcal{A}\ln(q/q^*), \tag{24}$$

$$\frac{\dot{q}}{q} = \frac{d\ln(q/q^*)}{dt} = \mathcal{B}\ln(k/k^*) + \mathcal{C}\ln(q/q^*), \tag{25}$$

where

$$\mathcal{A} = \frac{q^*}{b\omega}[(q^*-1)/b]^{(1/\omega)-1} > 0, \tag{26}$$

$$\mathcal{B} = -\frac{f''(k^*)k^*}{q^*} > 0, \tag{27}$$

$$\mathcal{C} = \frac{f'(k^*)}{q^*} - \frac{b^{-1/\omega}\omega/(1+\omega)}{q^*}(q^*-1)^{1/\omega}\left(1 + \frac{q^*}{\omega}\right) = r - x - n > 0, \tag{28}$$

so that $\mathcal{A}$ and $\mathcal{C}$ can be directly expressed in terms of given parameter values. The determination of $\mathcal{B}$ depends on a particular production function and will be discussed later. The Jacobian matrix $\begin{pmatrix} 0 & \mathcal{A} \\ \mathcal{B} & \mathcal{C} \end{pmatrix}$ has a negative determinant, $-\mathcal{A}\mathcal{B}$. Therefore, the log-linearized system exhibits a saddle-path stability – the matrix has one positive and one negative eigenvalue. These eigenvalues are the solutions to the quadratic characteristic equation:

$$\epsilon_{1,2} = \frac{r - x - n \pm \sqrt{(r-x-n)^2 + 4\mathcal{A}\mathcal{B}}}{2}, \tag{29}$$

where $\epsilon_1 > 0$ and $\epsilon_2 < 0$. The general solution to (24) and (25) is then

$$\ln(k/k^*) = \nu_{11}e^{\epsilon_1 t} + \nu_{12}e^{\epsilon_2 t}, \tag{30}$$

$$\ln(q/q^*) = \nu_{21}e^{\epsilon_1 t} + \nu_{22}e^{\epsilon_2 t}. \tag{31}$$

Because $\epsilon_1 > r - x - n > 0$, this root represents explosive paths and consequently must be excluded for the transversality condition (13) to be satisfied. Therefore:

$$\ln(k/k^*) = \nu_{12}e^{\epsilon_2 t} = \ln(k_0/k^*)e^{\epsilon_2 t}, \tag{32}$$

$$\ln(q/q^*) = \nu_{22}e^{\epsilon_2 t} = \nu \ln(k_0/k^*)e^{\epsilon_2 t}, \tag{33}$$

where $(1, \nu)$ is the eigenvector corresponding to ϵ_2.

4 Convergence

In empirical studies, the rate of convergence (convergence coefficient) is typically assumed to be constant. This constancy is justified by the log-linearized version of the basic neoclassical model and constitutes a good approximation in the neighborhood of the steady state. In the present model, the rate of convergence is constant for physical capital in the log-linearized approximation, and it is constant for output in this approximation if the production function takes a Cobb-Douglas form. However, for more general production specifications, the rate of output convergence differs from that of capital convergence if the economy is not in the steady state. Moreover, convergence speed is no more constant in the exact model (either for capital or output) outside the steady state, even if the production function is Cobb-Douglas.

In order to describe the situations in which the convergence speed changes, we introduce two related concepts of the convergence rate (both for capital and output). First, in order to indicate how the growth rate of capital *locally* reacts to changes in the level of capital, we define one convergence coefficient for k as

$$\beta_k \equiv -\frac{d\gamma_k}{d(\ln k)}, \tag{34}$$

where

$$\gamma_k \equiv \frac{\dot{k}}{k}. \tag{35}$$

Second, so as to describe the *global* tendency of the economy to grow faster the further below its steady state the economy is, we define another convergence coefficient as

$$\tilde{\beta}_k \equiv -\frac{\gamma_k}{\ln(k/k^*)}, \tag{36}$$

where $\tilde{\beta}_k$ is extended by its limit if $k = k^*$. Note that β_k and $\tilde{\beta}_k$ coincide if the time evolution of capital is described by (32). In this case

$$\tilde{\beta}_k = \beta_k = -\epsilon_2 > 0. \tag{37}$$

Similar definitions can be written for output convergence rates:

$$\beta_y \equiv -\frac{d\gamma_y}{d(\ln y)}, \tag{38}$$

where

$$\gamma_y \equiv \frac{\dot{y}}{y}, \tag{39}$$

and

$$\tilde{\beta}_y \equiv -\frac{\gamma_y}{\ln(y/y^*)}, \tag{40}$$

where $\tilde{\beta}_y$ is again assumed to be equal to its limit if $y = y^*$ (y^* is the steady-state level of output per effective worker). Unlike β_k and $\tilde{\beta}_k$ (which are equal in this log-linearized approximation), β_y and $\tilde{\beta}_y$ may differ and change over time if we assume a general CRTS production function. In the steady state, however, both β_y and $\tilde{\beta}_y$ equal β_k:

$$(\tilde{\beta}_y)_{y=y^*} = \lim_{k \to k^*} \tilde{\beta}_k \frac{\ln(k/k^*) f'(k)k/f(k)}{\ln[f(k)/f(k^*)]} = \beta_k, \tag{41}$$

which follows from

$$\left(\frac{d\ln[f(k)]}{d(\ln k)}\right)_{k=k^*} = \lim_{k \to k^*} \frac{\ln[f(k)] - \ln[f(k^*)]}{\ln k - \ln k^*}. \tag{42}$$

L'Hospital's rule implies that β_y converges to $\tilde{\beta}_y$ if y tends to y^*. Let β denote this common steady-state value. From (29) and (37) it follows that

$$\beta = \frac{\sqrt{(r-x-n)^2 + 4\mathcal{A}\mathcal{B}} - (r-x-n)}{2}. \tag{43}$$

5 Production Function Specifications

So far we have not imposed any additional restriction on the production function. At this point, we consider first the Cobb-Douglas specification. Then, we extend the specification to a more general constant elasticity of substitution (CES) production function. We investigate how the elasticity of substitution between labor and capital influences convergence.

5.1 Cobb-Douglas Specification

Equation (1) takes the form

$$Y = AK^{\alpha}(Le^{xt})^{1-\alpha}, \tag{44}$$

where $0 < \alpha < 1$, and A is a fixed technological parameter. Equation (8) can be rewritten as

$$y = f(k) = Ak^{\alpha}. \tag{45}$$

The formula for $\mathcal{B}$ can be expressed as

$$\mathcal{B} = (1-\alpha)\frac{f'(k^*)}{q^*}, \tag{46}$$

so that $\mathcal{B}$ (and, therefore, β) can be expressed in terms of given parameter values [see equation (23)].

The Cobb-Douglas function can easily be proven to satisfy two important properties:

Property 1: The Cobb-Douglas production function is the only type CRTS production function (increasing in both inputs) for which the capital share does not depend on k.

Proof: It follows directly from integrating $\frac{d\ln[f(k)]}{d(\ln k)} = const.$

Property 2: The Cobb-Douglas production function is the only type CRTS production function (increasing in both inputs) for which $\beta_y = \beta_k$ and $\tilde{\beta}_y = \tilde{\beta}_k$ for any k.

Proof: From the definitions (34) and (38) it can be derived that

$$\beta_y = \beta_k - \gamma_k \frac{f(k)}{f'(k)} \frac{d[f'(k)k/f(k)]}{dk}. \tag{47}$$

The equality $\beta_y = \beta_k$ holds identically if and only if the capital share is independent of k, i.e., if and only if the production function takes the Cobb-Douglas form. From the definitions (36) and (40) it follows that $\tilde{\beta}_k = \tilde{\beta}_y$ if and only if

$$\frac{d\ln[f(k)]}{d(\ln k)} = \frac{\ln[f(k)] - \ln[f(k^*)]}{\ln k - \ln k^*} \tag{48}$$

for any $k \neq k^*$. If we introduce the substitutions $\tilde{x} = \ln k$, $\tilde{x}_0 = \ln k^*$, and $G(\tilde{x}) = \ln[f(k)]$, this equality takes the form

$$\frac{dG(\tilde{x})}{G(\tilde{x}) - G(\tilde{x}_0)} = \frac{d\tilde{x}}{\tilde{x} - \tilde{x}_0}. \tag{49}$$

Integrating this with respect to $\tilde{x}$ leads to $G(\tilde{x}) = G(\tilde{x}_0) + const.(\tilde{x} - \tilde{x}_0)$, so that $f(k) = f(k^*)(k/k^*)^{const.}$. Note that the exponent of (k/k^*) must lie between 0 and 1 if production is increasing in both inputs (K and L). The production function is therefore Cobb-Douglas, Q.E.D.

5.2 CES Specification

Assume now a more general production function:

$$Y = F(K, Le^{xt}) = \tilde{A}\ [uv^{\psi}K^{\psi} + (1-u)(1-v)^{\psi}(Le^{xt})^{\psi}]^{1/\psi}, \tag{50}$$

where $\tilde{A}$ is a fixed technological parameter, $0 < u < 1$, $0 < v < 1$, and $\psi < 1$ are constants, and $\xi \equiv 1/(1-\psi)$ is the elasticity of substitution between K and L. The intensive form of this production function is

$$y = f(k) = \tilde{A}\ [u\ v^{\psi}k^{\psi} + (1-u)(1-v)^{\psi}]^{1/\psi}. \tag{51}$$

This CES (constant elasticity of substitution) function approaches the Cobb-Douglas function $y = \tilde{A}\ v^u(1-v)^{1-u}k^u$ if $\psi \to 0$;[9] in this case, the capital share is constant and equal to u. If $\psi \neq 0$, the capital share is no more constant during the transition; however, it is constant in the steady state. Let us calibrate the CES function in such a way that the steady-state capital share is equal to u for any value of ψ.[10] This calibration makes it possible to insulate the effect of the elasticity of substitution from that of the capital share. The capital share for the CES function is

$$f'(k)k/f(k) = [uv^{\psi}k^{\psi} + (1-u)(1-v)^{\psi}]^{-1}uv^{\psi}k^{\psi}. \tag{52}$$

This capital share is equal to u in the steady state if

$$k^* = (1-v)/v, \tag{53}$$

[9] The proof follows directly from l'Hospital's rule.

[10] This calibration excludes endogenous growth which might otherwise occur for a sufficiently high ψ.

which should be viewed as a condition for v. This calibration implies

$$f(k^*) = \tilde{A}\frac{k^*}{1+k^*} = \frac{f'(k^*)k^*}{u}, \tag{54}$$

$$f(k) = f(k^*)[u(k/k^*)^{\psi} + 1 - u]^{1/\psi}, \tag{55}$$

$$f'(k^*) = \tilde{A}\ uv = \tilde{A}\ u\frac{1}{1+k^*}, \tag{56}$$

$$f'(k) = f'(k^*)[u + (1-u)(k^*/k)^{\psi}]^{1/\psi - 1}, \tag{57}$$

$$f''(k^*) = -\tilde{A}\ u\ (1-u)(1-\psi)v^2/(1-v). \tag{58}$$

This enables us to express $\mathcal{B}$ [see equation (27)] in terms of $f'(k^*)$:

$$\mathcal{B} = (1-u)(1-\psi)\frac{f'(k^*)}{q^*} = \frac{(1-u)f'(k^*)}{\xi q^*}. \tag{59}$$

Thus, $\mathcal{B}$ here looks similar to the Cobb-Douglas case [see equation (46)] if one replaces α by u and introduces a new term, ξ. The higher the elasticity of substitution between labor and capital, ξ, the lower $\mathcal{B}$, and the lower the convergence coefficient, β.[11] The economic interpretation of this finding is clear: if labor and capital are good substitutes in production, the rate of return on capital does not react much to the capital-labor ratio. Consequently, capital returns and investment do not tend to be very high in economies with low capital intensity, i.e., convergence is slow. It is also interesting to note that the same result for $\mathcal{B}$ is obtained if we consider the Kmenta approximation of the CES function. The Kmenta approximation arises from Taylor's expansion of the logarithm of the CES function with respect to ψ around the point $\psi = 0$ (corresponding to the Cobb-Douglas function) and taking only the first element containing ψ.[12] The Kmenta approximation of the CES function (51) satisfies

$$\ln[f(k)] = \ln\tilde{A} + u\ln k + u\ln v + (1-u)\ln(1-v) + \frac{1}{2}\psi u(1-u)\left[\ln\left(\frac{kv}{1-v}\right)\right]^2. \tag{60}$$

Letting the capital share be equal to u in the steady state again leads to (53), from which follows, after some algebra, the expression (59) for $\mathcal{B}$.

[11] The high elasticity of substitution is not necessarily unrealistic. As Mankiw [13] argues, for instance, the elasticity of substitution between capital and labor may be effectively large due to the effects of international trade and specialization.

[12] See Kmenta [11], p. 514. In the present context, we assume a bit more general CES specification, and the Kmenta proxy is modified correspondingly.

Unless the CES function reduces to the Cobb-Douglas function, the capital share changes with k, and β_y and $\tilde{\beta}_y$ are not equal to β during the transition. Substituting (53) in (52) yields

$$f'(k)k/f(k) = \frac{u\kappa}{u\kappa + 1 - u}, \tag{61}$$

where

$$\kappa \equiv (k/k^*)^{\psi}. \tag{62}$$

Therefore, the capital share depends positively (negatively) on k if $\psi > 0$ ($\psi < 0$). The output convergence coefficients can be shown to satisfy

$$\beta_y = \beta_k + \tilde{\beta}_k \frac{\ln \kappa}{1 + \kappa u/(1-u)}, \tag{63}$$

$$\tilde{\beta}_y = \tilde{\beta}_k u \; \frac{\kappa \ln \kappa}{[u\kappa + (1-u)] \ln[u\kappa + (1-u)]}. \tag{64}$$

6 Parameter Specifications

Let us adopt the following baseline specification: $r = 0.06$/year, $x = 0.02$/year, $n = 0.01$/year, $\delta = 0.05$/year, and $\omega = 1$. In addition, let us assume the Cobb-Douglas production with the capital share $\alpha = 0.75$. This specification corresponds to that used by Barro and Sala-i-Martin [1] in Section 3.5. $\mathcal{C}$ is then equal to 0.03/year, and $\mathcal{A}$, $\mathcal{B}$, and β can be determined for given q^* [note that b satisfies (22)].

In accordance with Barro and Sala-i-Martin [1], we conclude that this specification cannot be consistent with the empirically observed slow convergence of about 2%/year unless the costs of adjustment are *extremely* large. The first rows of Tables 1, 2, and 3 all correspond to the baseline specification; β tends to 2.5%/year only if q^* tends to infinity. Capital is assumed to be a composite of human and physical capital. The q values for physical capital are likely to be small.[13] Even if we make a realistic assumption of the large adjustment costs for changing human capital, there is no reason to expect the q^* values of the composite capital to be higher than, say, 2 or 3. To achieve slower convergence, we should deviate from the baseline specification.

[13] The empirical estimates of q are typically well below 1.5 (Blanchard et al. [4], Hayashi [9]).

Let us make perturbations from the baseline specification and investigate how the rate of convergence, β, depends on various parameters in the neighborhood of the steady state: the rate of convergence is not very sensitive to r; it depends slightly positively on r if q^* is low ($q^* < 1.4$) and slightly negatively on r if q^* is large. The coefficient β depends positively on x, δ, and n, but it reacts less than proportionately to each of these parameters. Finally, β depends negatively on ω and α.

Let us consider three kinds of departures from the baseline specification. First, let $\omega > 1$. The coefficient ω indicates how quickly the adjustment costs rise with the investment-capital ratio. The investment-capital ratio is higher the lower the economy is below its steady state. Therefore, a higher ω slows down convergence. Table 1 illustrates how β depends on ω and q^* for the baseline values of other parameters. Second, let the capital share, α, be higher than 0.75. A higher capital share pushes the model closer to its endogenous limit and decreases the speed of convergence. Table 2 shows how β depends on α and q^* for the baseline values of other parameters. Third, let us assume a CES production specification with the same steady-state capital share and a higher elasticity of substitution between labor and capital. Table 3 indicates how β depends on the elasticity of substitution, ξ, and on q^* if other parameters are held at their baseline values (including $u = 0.75$, which corresponds to $\alpha = 0.75$).

7 Conclusion

The basic neoclassical model leads to counterfactual implications when applied to open economies with capital mobility. Despite substantial progress in recent research, this problem has not been solved yet to complete satisfaction. In this paper, we analyze quantitatively the effects of production-function and adjustment-cost parameters on the speed of convergence in a model with no borrowing restrictions. We find a closed-form solution for the convergence coefficient in the log-linearized approximation if the production function takes a relatively general CES form, and we quantify the conditions leading to sufficiently slow convergence.

Table 1: Convergence coefficients β (in %/year) for different combinations of ω and q^* and the baseline values of other parameters.

$\omega \setminus q^*$	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	3.0	4.0	∞
1	13.9	9.7	8.0	6.9	6.3	5.8	5.4	5.1	4.9	4.7	3.7	3.4	2.5
2	9.4	6.4	5.2	4.5	4.0	3.6	3.4	3.2	3.0	2.8	2.2	1.9	1.3
3	7.4	5.0	4.0	3.4	3.0	2.7	2.5	2.3	2.2	2.1	1.5	1.3	0.9
4	6.2	4.2	3.3	2.8	2.4	2.2	2.0	1.9	1.8	1.7	1.2	1.0	0.6
5	5.4	3.6	2.8	2.4	2.1	1.9	1.7	1.6	1.5	1.4	1.0	0.8	0.5

Table 2: Convergence coefficients β (in %/year) for different combinations of α and q^* and the baseline values of other parameters.

$\alpha \setminus q^*$	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	3.0	4.0	∞
0.75	13.9	9.7	8.0	6.9	6.3	5.8	5.4	5.1	4.9	4.7	3.7	3.4	2.5
0.80	12.3	8.6	7.0	6.1	5.5	5.0	4.7	4.5	4.2	4.1	3.2	2.9	2.2
0.85	10.4	7.3	5.9	5.1	4.6	4.2	3.9	3.7	3.5	3.4	2.7	2.4	1.8
0.90	8.3	5.7	4.6	4.0	3.5	3.2	3.0	2.8	2.7	2.6	2.0	1.8	1.3
0.95	5.5	3.7	2.9	2.5	2.2	2.0	1.9	1.7	1.7	1.6	1.2	1.1	0.7

Table 3: Convergence coefficients β (in %/year) for different combinations of ξ and q^* and the baseline values of other parameters.

$\xi \setminus q^*$	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	3.0	4.0	∞
1	13.9	9.7	8.0	6.9	6.3	5.8	5.4	5.1	4.9	4.7	3.7	3.4	2.5
2	9.4	6.5	5.3	4.6	4.1	3.8	3.5	3.3	3.1	3.0	2.3	2.1	1.5
3	7.5	5.1	4.1	3.5	3.1	2.9	2.7	2.5	2.4	2.3	1.8	1.6	1.1
4	6.3	4.3	3.4	2.9	2.6	2.4	2.2	2.1	1.9	1.9	1.4	1.3	0.9
5	5.5	3.7	2.9	2.5	2.2	2.0	1.9	1.7	1.7	1.6	1.2	1.1	0.7

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